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ENGINEERING MECHANICS
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Printed Notes
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Difference between CENTRE OF GRAVITY and CENTROID:

- The term centre of gravity applies to bodies with mass and weight, and centroid applies to plane areas.
- Centre of gravity of a body is the point through which the resultant gravitational force (weight) acts for any orientation of body whereas centroid is the point in the plane area such that the moment of that area about any axis through that point is zero.

MOMENT OF INERTIA

Moment of inertia is a purely mathematical term which gives a quantitative estimate of the relative distribution of the area with respect to some reference axis.

Consider a thin lamina of area A as shown in figure. Let dA be an elemental area in the plane figure.

Let ‘x’ be the distance of the elemental area dA from Y-axis and ‘y’ be the distance of elemental area dA from X-axis.

The moment of inertia of the area about X-axis, \( I_{XX} = \sum dA \cdot y^2 \)

\( dA \cdot y \) is known as the first moment of area about Y-axis. The first moment of area is used to determine the centroid of the area.

If the moment of area is again multiplied by the perpendicular distance between dA and Y-axis, then the quantity dAy^2 is known as moment of moment of area or second moment of area or area moment of inertia about Y-axis.

Similarly,

\( I_{YY} = \sum dA \cdot x^2 \)

In general, If an elemental area ‘dA’ is considered and ‘r’ is the distance of the elemental area from a reference axis AB,
Then the moment of inertia of the entire area about reference axis is given by

\[ I_{AB} = \sum dA \cdot r^2 = \int dAr^2 \]

If instead of area, the mass (m) of the body is taken into account, then the second moment is known as *second moment of mass* or **mass moment of inertia**.

∴ if \( m \) is the mass of the body and \( x \) and \( y \) are the perpendicular distances of its centre of gravity from \( Y \) and \( X \) axes,

Mass moment of inertia about \( Y \) axis = \( mx^2 \)

Mass moment of inertia about \( X \) axis = \( my^2 \)

“Hence the product of area(or mass) and the square of the distance of the centroid (or centre of gravity) of the area(or mass) from an axis is known as Moment of Inertia of the area(or mass) about that axis.”

Moment of Inertia is represented by \( I \). Moment of inertia about \( X \)-axis is represented by \( I_{XX} \) and about \( Y \)-axis is represented as \( I_{YY} \).

If \( G \) is the centroid of the body, the axis passing through the centroid of the body is known as centroidal axis and the moment of inertia about the centroidal axis is given by \( I_G \).

Since it is a term obtained by multiplying area by the square of the distance, its unit in SI is given as \( m^4 \).

**POLAR MOMENT OF INERTIA**
Moment of inertia about an axis perpendicular to the plane of an area is known as polar moment of inertia. It is denoted by $J$ or $I_{zz}$. Thus the moment of inertia about an axis perpendicular to the plane of the area at O in the figure is called polar moment of inertia at O and is given by,

$$I_{zz} = \sum dA \cdot r^2$$

**RADIUS OF GYRATION**

Consider an area which has a moment of inertia $I$ with respect to a reference axis AB.

Let us assume that this area is compressed to a thin strip of negligible width parallel to axis AB. For this strip to have the same moment of inertia $I$ with respect to the same reference axis AB, the strip should be placed at a distance “$k$” from the axis AB such that $I = Ak^2$, where $k = \sqrt{I/A}$ is called the radius of gyration.

Radius of gyration of a body or a given area about an axis is a distance such that its square multiplied by the area gives moment of inertia of the area about the given axis. It is a mathematical term given by the relation

$$k = \sqrt{I/A},$$

where $k$ = radius of gyration

$I$ = Moment of Inertia

$A$ = Cross sectional area.

Suffixes with the moment of inertia also accompany the term radius of gyration $k$.

**ie**

$$k_{xx} = \sqrt{(I_{xx}/A)}$$

$$k_{yy} = \sqrt{(I_{yy}/A)}$$
PERPENDICULAR AXIS THEOREM

“The moment of inertia of an area about an axis perpendicular to its plane at any point O is equal to the sum of moments of inertia about any two mutually perpendicular axis through the same point lying in the plane of the area”

If \( I_{XX} \) and \( I_{YY} \) be the moment of inertia of a plane area about two mutually perpendicular axis \( X-X \) and \( Y-Y \) in the plane of the area, then the moment of inertia of the area \( I_{ZZ} \) about the axis \( Z-Z \) is given by

\[ I_{ZZ} = I_{XX} + I_{YY} \]

\( I_{ZZ} \) is also known as polar moment of inertia.

Proof:

Let a plane area \( A \) is lying in plane \( X-Y \) is shown in figure.

Let \( x \) = distance of \( dA \) from \( Y \) axis
\( y \) = distance of \( dA \) from \( X \) axis
\( r \) = distance of \( dA \) from \( Z \) axis.

Then \( r^2 = x^2 + y^2 \)

Now moment of inertia of \( dA \) about \( X \)-axis = \( dA \times y^2 \)
moment of inertia of total area \( A \) about \( X \)-axis = \( \sum dA \times y^2 \)
moment of inertia of entire area \( A \) about \( Y \)-axis = \( \sum dA \times x^2 \)
moment of inertia of total area \( A \) about \( Z \)-axis = \( \sum dA \times r^2 \)

\[ \sum dA \times r^2 = \sum dA (x^2 + y^2) \]
\[ = \sum dA \times x^2 + \sum dA \times y^2 \]
\[ = I_{XX} + I_{YY} \]

\[ I_{ZZ} = I_{XX} + I_{YY} \]
PARALLEL AXIS THEOREM

"If the moment of inertia of a plane area about an axis in the plane of an area through the C.G of the plane area be represented by \( I_G \), then the moment of inertia of the given plane area about a parallel axis AB in the plane of area at a distance ‘\( h \)’ from the C.G is given by

\[
I_{AB} = I_G + Ah^2
\]

Proof:

Consider an elemental parallel strip dA at a distance y from centroidal axis,

\[
\text{Then } I_{AB} = \sum (y+h)^2 \cdot dA
\]

\[
= \sum (y^2 + 2yh + h^2) \cdot dA
\]

\[
= \sum (y^2) \cdot dA + \sum (2yh) \cdot dA + \sum (h^2) \cdot dA
\]

Here, 1. \( \sum (y^2) \cdot dA = I_G \).

2. \( \sum (2yh) \cdot dA = 2h \cdot \sum y \cdot dA \), where \( \sum y \cdot dA \) is the moment of the total area about X-X axis, which is equal to zero because X-X is the centroidal axis.

3. \( \sum (h^2) \cdot dA = h^2 \cdot \sum dA = h^2 \cdot A \)

Therefore \( I_{AB} = I_G + Ah^2 \)
### Moment of Inertia Formulas

**Note:** In the table below, \( I_{Gx} \) and \( I_{Gy} \) indicates the moment of inertia is taken about axes that passes through the centroid, denoted as ‘\( G \)’. \( I_{AB} \) denotes the moment of inertia about base \( AB \).

#### Rectangle:
\[
\begin{align*}
I_{Gx} &= \frac{1}{12} bd^3 \\
I_{Gy} &= \frac{1}{12} db^3 \\
I_{AB} &= \frac{1}{3} bd^3 \\
Area &= bd
\end{align*}
\]

#### Triangle:
\[
\begin{align*}
I_{Gx} &= \frac{1}{36} bh^3 \\
I_{AB} &= \frac{1}{12} bh^3 \\
Area &= \frac{1}{2} bh
\end{align*}
\]

#### Circle:
\[
\begin{align*}
I_{Gx} &= I_{Gy} = \frac{1}{4} \pi R^4 \\
Area &= \pi R^2
\end{align*}
\]

#### Semi-circle:
\[
\begin{align*}
I_{Gx} &= \left( \frac{\pi}{8} - \frac{8}{9\pi} \right) R^4 = 0.11R^4 \\
I_{Gy} &= \frac{1}{8} \pi R^4 \\
I_{AB} &= \frac{1}{8} \pi R^4 \\
Area &= \frac{\pi R^2}{2}
\end{align*}
\]

#### Quarter Circle:
\[
\begin{align*}
I_{Gx} &= \left( \frac{\pi}{16} - \frac{4}{9\pi} \right) R^4 = 0.055R^4 \\
I_{AB} &= \frac{1}{16} \pi R^4 \\
Area &= \frac{\pi R^2}{4}
\end{align*}
\]