THE PROPERTY OF BODIES BY VIRTUE OF WHICH A FORCE IS EXERTED BY A STATIONARY BODY ON THE MOVING BODY IS CALLED FRICTION.

ACTS OPPOSITE TO THE DIRECTION OF MOTION.

ACTS PARALLEL TO THE SURFACE OF CONTACT.

**FRICTION CHARACTERISTICS**

![Graph showing static and dynamic friction](image)

**TYPES OF FRICTION**

i) **Static friction**: If two surfaces, which are in contact are at rest, the force experienced by one surface is called static friction.

ii) **Dynamic friction**: If one surface starts moving and the other is at rest, the force experienced by the moving surface is called dynamic friction.
(iii) Solid friction / Dry friction: If blow two surfaces no lubrication is used the friction is called dry friction.

(iv) Fluid friction: It is the friction blow the adjacent layers of fluid, moving at different velocities.

A body of mass 10 kg resting on a horizontal plane as shown in fig. The coefficient of friction blow the body + plane is 0.2. Determine the minimum value of P required to move the body towards right.

\[ \sum F_x = 0 \]
\[ P - f = 0 \]
\[ f = \mu N \]
\[ P - \mu N = 0 \] — (i)
To find $N$

\[ \Sigma F_y = 0 \]
\[ N - W = 0 \]
\[ N = 10 \times 9.81 \]
\[ N = 98.1 \text{ N} \]

(1) \Rightarrow

\[ P - (0.2 \times 98.1) = 0 \]
\[ P = 19.62 \text{ N} \]

? A body of mass $150 \text{ N}$ resting on a horizontal plane is subjected to a force as shown in figure. Determine the minimum value of $P$ to cause the motion.

A. FBD

\[ \Sigma F_x = 0 \]
\[ P \cos 20^\circ - f = 0 \]
\[ 0.94 P - f = 0 \quad (i) \]
\[ 0.94 P - \mu N = 0 \quad (ii) \]
\[ \Sigma f_y = 0 \]
\[-P \sin 20^\circ + N - W = 0. \]
\[-0.34 P + N = 150. \]

\[ N = 150 + 0.34 P. \] — (2)

Substitute (2) in (1)
\[ 0.94 P - \mu (150 + 0.34 P) = 0 \]
\[ 0.94 P - 0.3 (150 + 0.34 P) = \]
\[ 0.94 P - 45 - 0.102 P \]
\[ 0.838P = 45 \]
\[ P = 53.7 N \]

**Laws of Solid Friction (Coulomb/Dry Friction)**

> The force of friction acts in the opposite direction in which the surface is having a tendency to move.
> The force of friction is equal to the force applied to the surface so long as the surface is at rest.
> When the surface is at the point of motion, the force of friction is maximum and this maximum frictional force is called the limiting frictional force.
> The limiting frictional force bears a constant ratio to the normal reaction between two surfaces.
> The limiting force of friction does not depend upon the shape and areas of the surfaces in contact.
> The ratio between the limiting friction and normal reaction is slightly less when two surfaces are in motion.
> The force of friction is independent of the velocity of sliding.
Determine the coefficient of friction between the body and the plane if a force of 1000 N causes the motion of the body towards right.

A. FBD

\[ \sum F_x = 0, \]
\[ 1000 \cos 15^\circ - F = 0 \]
\[ F = 965.9 \text{ N} \]

\[ \sum F_y = 0, \]
\[ -1000 \sin 15^\circ - W + N = 0 \]
\[ -258.8 - 147 + N = 0 \]
\[ N = 405.8 \text{ N} \]

\[ F = \mu N \]
\[ \mu = \frac{F}{N} = \frac{965.9}{405.8} = 2.38 \]
For bodies resting on inclined plane, component of weight parallel to plane is $w \sin \alpha$ and component of weight $W$ to plane is $w \cos \alpha$ where $\alpha$ is the inclination of plane.

A block of weight 200 N is resting on a plane of inclination 40°. Determine the minimum force required to move the body upward along the plane.

**FBD**

\[ \sum F_x = 0. \]

\[ P - F - W \sin 40° = 0. \]

\[ P - \mu N = 188.5 \quad (1) \]
\[ \Sigma F_y = 0. \]

\[ N - H \cos \alpha = 0 \]

\[ N = 153.2 \text{ N} \]

\[ (i) \quad P - 0.1 \times 153.2 = 128.5 \]

\[ P = 143.82 \text{ N} \]

A body of mass 12 kg resting on an inclined plane is shown in figure. Determine the minimum force \( P \) required to move the body along the plane.

\[ \Sigma F_x = 0. \]

\[ P \cos 15^\circ - \mu \cdot 35 \cdot P = 0. \]

\[ 0.96 P - 67.5 - \mu N = 0. \]
\[ 2 \times 15^\circ \cos 75^\circ \text{ - } 7 \text{ kg} \cos 35^\circ + N = 0 \]
\[ -0.26P - 96.43 + N = 0 \]
\[ N = 0.26P + 96.43 \]
\[ 0.96P - 0.2(0.26P + 96.43) = 67.5 \]
\[ 0.96P - 0.052P - 19.29 = 67.5 \]
\[ 0.908P = 86.79 \]
\[ P = 95.58 \text{ N} \]

**Motion of Connected Bodies**

A block of mass 10 kg is connected to another block of mass 7 kg, as shown in figure. Determine the coefficient of friction between the table and 10 kg of body.

**A. FBD of A**

\[ F \]
\[ N \]
\[ T \]
\[ \sum F_x = 0 \]
\[ q - F = 0 \]
\[ q - N = 0 \]  \quad (1)

\[ \sum F_y = 0 \]
\[ N - w = 0 \]
\[ N = 98.1 \text{ N} \]
\[ q - 98.1 \mu = 0 \]  \quad (2)

\[ \text{FBD of B} \]

\[ \sum F_y = 0 \]
\[ T - w = 0 \]
\[ T = 68.67 \text{ N} \]
\[ \mu = \frac{68.67}{98.1} = 0.7 \]
\[ \mu = 0.7 \]
### 3. Determination of Moment of Inertia

Determine the moment of inertia of the given section. A tripod is shown in figure. It is required to determine the force acting on its legs; if the resultant force is zero:

1. Determine the centroid of the following area:

   - Ax = 30.4 cm
   - Ay = 19.0 cm

2. Determine the moment of inertia of the given section:

   - $I_x = 2667.3$ cm$^4$
   - $I_y = 26013.2$ cm$^4$

3. Determine the area moment of inertia with its centroidal axis.

### Table: Moment of Inertia Calculation

| No. | $A_i$ | $x_i$ | $y_i$ | $A_i(x_i - x)^2$ | $A_i(y_i - y)^2$ | Total
<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>1135022.6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1854.5</td>
<td>76.51</td>
<td>7.5</td>
<td>606.80</td>
<td>45380.00</td>
<td>41541.8</td>
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<tr>
<td>3</td>
<td>219604.1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

### Calculation Details

- $x = \frac{30.4 + 19.0}{2} = 24.7$ cm
- $y = \frac{2667.3 + 26013.2}{2} = 14340.25$ cm

### Notes

- The centroidal axes are $x$ and $y$.
- The moment of inertia is calculated using the parallel axis theorem.
- The table shows the individual moments of inertia and their contributions to the total moment of inertia.
\[ I_{x-x} = 355807.5 \text{ cm}^4 \]

\[ I_{y-y} = 1313982.09 \text{ cm}^4 \]

<table>
<thead>
<tr>
<th>No.</th>
<th>( A_i )</th>
<th>( x_i )</th>
<th>( y_i )</th>
<th>( A_i x_i )</th>
<th>( A_i y_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>96.</td>
<td>4</td>
<td>6</td>
<td>384</td>
<td>576</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>2.7</td>
<td>1.3</td>
<td>10.8</td>
<td>5.2</td>
</tr>
<tr>
<td>2</td>
<td>12.</td>
<td>3</td>
<td>4.8</td>
<td>48</td>
<td>15.6</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>5.3</td>
<td>1.3</td>
<td>42.4</td>
<td>10.4</td>
</tr>
<tr>
<td>2</td>
<td>25.13</td>
<td>4</td>
<td>10.3</td>
<td>100.5</td>
<td>258.8</td>
</tr>
<tr>
<td>Total</td>
<td>58.87</td>
<td></td>
<td></td>
<td>230.3</td>
<td>301.6</td>
</tr>
</tbody>
</table>

\[ \bar{x} = 3.89 \text{ cm} \]

\[ \bar{y} = 5.12 \text{ cm} \]
Block A weigh 13 N is resting on a rough inclined plane as shown in figure. The block is tied up by a horizontal string which has a tension of 5 N.

Determine,

i) friction on the block

ii) Normal reaction by the inclined plane.

iii) Coefficient of friction bluff body & plane.

\[ \Sigma F_x = 0 \]

\[ 0 = 13 \cos 40^\circ - N \sin 40^\circ + T \sin 40^\circ + f = 0 \]

\[ 0.77 T - 9.6 + \mu N = 0 \]

\[ \mu N = 5.75 \] (i)

\[ \Sigma F_y = 0 \]

\[ N - 13 \cos 40^\circ - T \sin 40^\circ = 0 \]

\[ N = 11.5 - 3.2 = 0 \]

\[ N = 14.7 \text{ N} \]

\[ \mu = 0.4 \]
A string connects two bodies of weight 400 N and 800 N. They are placed on an inclined plane and string is parallel to the plane. The coefficient of friction between body A and plane is 0.15 and that for body B is 0.4. Determine the inclination of plane to the horizontal and the tension in the string when the motion is about to take place. The body weighing 400 N is below the second body.

A. FBD of A

\[ \Sigma F_x = 0 \]
\[ T - W \sin \alpha + F = 0 \]
\[ T - 400 \sin \alpha + 0.15 N = 0 \quad (1) \]

\[ \Sigma F_y = 0 \]
\[ N - W \cos \alpha = 0 \]
\[ N - 400 \cos \alpha = 0 \]
\[ N = 400 \cos \alpha \]
FBD of B

\[ T - 400\sin \alpha + 60 = 0 \]
\[ F = 400\sin \alpha - 60 \cos \alpha \]

\[ \sum F_x = 0 \]
\[ F - W\sin \alpha - T = 0 \]
\[ F = 800\sin \alpha - T = 0 \]
\[ 0.4N - 800\sin \alpha - T = 0 \]
\[ \sum F_y = 0 \]
\[ N - W\cos \alpha = 0 \]
\[ N = 800\cos \alpha \]
\[ 380\cos \alpha - 800\sin \alpha - T = 0 \]

\[ T = 665.57 \]
\[ F = 800\sin \alpha - 380\cos \alpha \]

\[ \sin \alpha = \frac{19}{50} \]
\[ \cos \alpha = \frac{\sqrt{21}}{5} \]

\[ -800\sin \alpha + 380\cos \alpha = 400\sin \alpha - 60\cos \alpha \]

\[ 1200\sin \alpha = 380\cos \alpha \]
\[ \tan \alpha = \frac{380}{1200} \]
\[ \alpha = 17.5^\circ \]
\[ T = 63.05 \text{N} \]
Determine the least value of $P$ to move the body towards right, assume the coefficient of friction under the block is 0.2 and pulley is frictionless.

A. FBD of 150 N

\[ \Sigma F_x = 0 \]
\[ T - 150 \sin 60^\circ - F = 0 \]
\[ T - 129.9 - 0.2 N_a = 0 \]
\[ \Sigma F_y = 0 \]
\[ N_a - 150 \cos 60^\circ = 0 \]
\[ N_a = 75 \text{ N} \]
\[ P = 144.9 \text{ N} \approx 145 \text{ N} \]
\[ \Sigma F_x = 0. \]
\[ P \cos \theta - T - P \sin \theta = 0. \]
\[ P \cos \theta - T = 0.2 N, N = 0. \]
\[ \Sigma F_y = 0. \]
\[ N_b + Psin \theta - U = 0. \]
\[ N_b + Psin \theta = 100. \]
\[ P \cos \theta - T = 0.2(100 - Psin \theta) = 0. \]
\[ P \cos \theta - 20 + 0.2 Psin \theta = 0. \]
\[ P \cos \theta - 165 + 0.2 Psin \theta = 0. \]
\[ P(\cos \theta + 0.2 \sin \theta) = 165 \]
\[ P = \frac{165}{\cos \theta + 0.2 \sin \theta} \]

When \( P \) is minimum denominator must be maximum.

IE, \((0.2 \sin \theta + \cos \theta)\) is maximum.

For this function to be maximum \( \frac{d}{d \theta} (0.2 \sin \theta + \cos \theta) = 0 \).

\[ \frac{d}{d \theta} (0.2 \sin \theta + \cos \theta) = 0. \]
\[ 0.2 \cos \theta - \sin \theta = 0. \]
\[ \tan \theta = 0.2 \]
\[ \theta = 11.31^\circ \]

\[ P = \frac{165}{\cos 11.31 + 0.2 \sin 11.31} = 161.8 \text{ N} \]
A block resting on a vertical plane is connected to another block of equal weight by a string as shown in figure. The string makes an angle of 45° with the horizontal. Determine the coefficient of friction between the block and the plane assuming it is same for all surfaces.

\[ \sum F_x = 0. \]

\[ N + T \cos 45° = 0. \]

\[ N = \frac{q}{\sqrt{2}} \quad (1) \]

\[ \sum F_y = 0. \]

\[ F - W - 9 \sin 45° = 0. \]

\[ \mu N = q \frac{1}{\sqrt{2}} - W = 0. \quad (2) \]

\[ W = - \mu q \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = - \frac{q}{\sqrt{2}} (\mu + 1) \]
\[ N = \frac{-q}{\sqrt{2}} (\mu + 1) \] — (A)

\[ \sum F_x = 0 \]
\[-q \sin 45^\circ \cdot L = 0 \]
\[-q \sin 45^\circ \cdot AN = 0 \]
\[ N = \frac{-q}{\mu \sqrt{2}} \]

\[ \sum F_y = 0 \]
\[ (100 \cdot 45^\circ) + N - 121 = 0 \]
\[ \frac{5}{\sqrt{2}} + \frac{q}{\mu \sqrt{2}} = \mu \cdot \sqrt{2} \]
\[ \mu N = \frac{q}{\mu \sqrt{2}} (\mu - 1) \] — (B)

Equating (4) + (5)
\[ \frac{-q}{\sqrt{2}} (\mu + 1) = \frac{q}{\sqrt{2}} \left( \frac{1}{\mu} + \frac{1}{\mu} \right) \]

\[ -\mu - 1 = \frac{\mu + 1}{\mu} \]
\[ -\mu^2 - \mu - \mu + 1 = 0 \]
\[ \mu^2 + 2\mu + 1 = 0 \]

\[ \mu = \frac{-2 \pm \sqrt{4 - 4}}{2} \]
\[ \mu = 1 + \frac{\sqrt{2}}{2} \]

\[ \mu = \frac{\sqrt{2}}{1} \text{ or } 1 \]

\[ \mu = 1 + \sqrt{2} \text{ or } 1 \]
\[
\frac{\mu}{(\mu + 1)} = \frac{\mu - 1}{-1 - \sqrt{2}}
\]

\[
\mu^2 + 2\mu - 1 = 0
\]

\[
\mu = -1 + \sqrt{2}, \quad -1 - \sqrt{2}
\]

**ANGLE OF REPOSE**

Angle of repose is the maximum inclination of plane at which a body resting on inclined plane is in equilibrium under the influence of friction only.

In the figure \( \alpha \) is angle of repose.

**Angle of friction**

It is the angle below the resultant of friction and normal reaction with the direction of plane. Angle of friction can also be defined as the angle that resultant makes with normal reaction.

In the figure 'P' is the external force applied.
on body, \( f \) is frictional force, \( N \) is normal reaction by the plane on the body and \( R \) is the resultant of \( f + N \). Then angle of friction is the angle below \( R + N \). It is denoted by \( \phi \).

From the figure,

\[
\tan \phi = \frac{f}{N} \quad f = \mu N, \quad \mu = \frac{F}{N}
\]

\[
\tan \phi = \mu
\]

Angle of friction can also be defined as an angle whose tangent gives coefficient of friction.

Relation btw Angle of Repose \( \phi \) Angle of Friction

Consider a body of weight \( \mu \) resting on a plane of inclination \( \alpha \) as shown in figure.

When \( \alpha \) is equal to angle of repose the forces acting on the block are in eqm.

Then, FBD of block.
\[ F_x = 0 \]
\[ F - N \sin \alpha = 0 \]
\[ F = N \sin \alpha \]
\[ F \sin \alpha + N \cos \alpha = 0 \]
\[ N = N \cos \alpha \] — (1)
\[ \tan \phi \cdot \tan \alpha \]
\[ \phi = \alpha \]

\[ \mu \cos \alpha = N \sin \alpha \]
\[ \mu = \tan \alpha \]

\[ \mu = \frac{\tan \phi}{\tan \alpha} \] — (2)

\[ \tan \phi \cdot \tan \alpha \]

\[ \phi = \alpha \]

ie Angle of repose = Angle of friction

A block A weighing 1000N rest on a rough inclined plane whose inclination to the horizontal is 50°. This block is connected to another block B weighing 1000N resting on a rough horizontal plane. By a weightless rigid bar inclined at 90° to the horizontal. Find the horizontal force required to be applied to block B just to move block A is upward direction. Assume angle of friction is 15°. All all surfaces.
A. FBD of body A

ΣFx = 0
- \( W \sin \alpha - F - 100 \cos \theta \)
- \( 100 \sin \alpha - \mu N - 0.94T = 0 \) — (1)

ΣFy = 0
\( N - W \cos \alpha + T \sin \theta \)
\( N - 100 \cos \alpha + 0.34T \)
\( N = 100 \cos \alpha - 0.34T \)
\( = 64.28 - 0.34T \)

\( -100 \sin \alpha - \mu (64.28 - 0.34T) - 0.94T = 0 \)

\( \alpha = 76.6^\circ - 64.28 \mu + 0.34 \mu T - 0.94T = 0 \)

\( \alpha = 93.95^\circ \)

\( \mu = \tan \phi \)
\( \mu = \tan 15^\circ \)
\( = 0.27 \)

FBD of B
Consider a ladder of weight \( H \) whose one end is placed on a horizontal plane and another end is supported on a vertical plane. The forces acting on the ladder are shown in the diagram.
A ladder of length 10 m is supported on a vertical plane with base of the ladder 6 m away from the vertical wall. Assuming a smooth wall, determine:

i) friction acting on point B

ii) normal reaction acting on point B

A. FBD of ladder

\[ \sum F_x = 0 \]
\[ N_A - N_f = 0 \]
\[ N_A = \mu N_B \] (1)

\[ \sum F_y = 0 \]
\[ N_B - W = 0 \]
\[ N_B = 100 N \]
\[ N_A = 100 \mu N \] (2)

Moment at B = 0.

Applying the resultant moment at point B = 0.
\[ \sum m_B = 0. \]

\( f_B + N_B \) do not produce any moment at point B

\[ (\omega \times BC) - (N_A \times OA) = 0. \]

\[ (100 \times 3) - (N_A \times 2) = 0. \]

\[ N_A = \frac{200}{8} = 25 \text{ N} \]

\[ (c) \Rightarrow \mu = \frac{N_A}{100} = 0.25 \]

\[ f_B = \mu N_B \]

\[ = 0.25 \times 100 \]

\[ = 25 \text{ N} \]

\[ N_B = 100 \text{ N}. \]

A ladder of weight 200 N is supported on a vertical wall as shown in figure. The coefficient of friction between the floor and the ladder is 0.2 and that between the wall and the ladder is 0.5. Determine how far a man of weight 500 N can climb along the ladder before it slips.
\[ \sum f_x = 0. \]
\[ N_A - f_B = 0. \]
\[ N_A = H N_B \quad (1) \]
\[ N_A = 0.2 N_B \quad (1) \]
\[ \sum f_y = 0. \]
\[ N_B - W + H \frac{m}{1} - f_A = 0. \]
\[ N_B - 200 - 500 + H N_A = 0. \]
\[ N_B + 0.15 N_A = 700. \]
\[ N_B + 0.15 (0.2 N_B) = 700. \]
\[ N_B + 0.03 N_B = 700. \]
\[ N_B = 679.6 \, N \]
\[ N_A = 135.92 \, N \]

Replace
\[ \sum M_B = 0. \]
\[ (H \cdot m \cdot BD) + (H \cdot X \cdot BC) - (f_A \cdot OB) - (N_A \cdot OA) = 0 \]
Assume the man has climbed $x$ m along the ladder.

\[
\cos 30^\circ = \frac{BD}{x} \\
BD = \frac{\sqrt{3}}{2} x
\]

\[
\cos 30^\circ = \frac{BC}{6} \\
BC = 3\sqrt{3} = 5.2 \text{ m}
\]

\[
250 \times 50 = 48250 \\
x = 0.028 \text{ m}
\]

A ladder of weight 200N is supported on a vertical wall as shown in figure. A person of weight 80N is climbing along the ladder. If coefficient of friction between ground and ladder is 0.4 and that between wall and ladder is 0.3. Determine how far along the ladder he can climb.

![Diagram of ladder and person climbing](image)
\[ \Sigma F_x = 0 \]

\[ NA - F_B = 0 \]

\[ NA = \mu NB, \quad (1) \]

\[ NA = 0.4 NB \]

\[ \Sigma F_y = 0 \]

\[ F_A - \mu m - \mu l + NB = 0 \]

\[ \mu NA - 700 + NB = 0 \]

\[ 0.3 (0.4 NB) + NB = 900 \]

\[ NB = \frac{900}{1.3} = 625 N \]

\[ NA = 850 N \]

\[ \Sigma M_B = 0 \]

\[ (\mu m \times BD) + (\mu l \times BC) - (F_A \times OB) - (NA \times OA) = 0 \]

\[ \cos 25 = \frac{BD}{x} \]

\[ \cos 25 = \frac{BC}{5} \]

\[ \cos 25 = \frac{OB}{10} \]

\[ \sin 25 = \frac{OA}{10} \]

\[ BD = 0.91 x \]

\[ BC = 4.58 \]

\[ OB = 9.06 \]

\[ OA = 4.23 \]
\[(500 \times 0.91x) + (200 \times 4.53) - (0.3 \times 250 \times 9.06) - (250 \times 4.23) = 0\]

\[4.55x = 8.81\]

\[x = 1.88\]

A uniform ladder of weight 150 N is shown in figure. Assume the coefficient of friction along all surfaces are 0.2. A weight of 400 N is placed on the ladder at a distance of 2 m from the top end. Determine the angle that ladder makes with horizontal when it is at the point of slipping.

\[\sum F_N = 0\]

\[N_A - F_B = 0\]

\[N_A = 0.2 \times N_B \quad \text{(1)}\]
\[ \sum F_y = 0 \]

\[ F_A + NB = 400 + 150 \]
\[ MA + NB = 550 \]
\[ OR(0.2NB) + NB = 550 \]

\[ NB = \frac{500}{1.04} = 480.78 \text{ N} \]

\[ NA = 90.15 \times 105.76 \text{ N} \]

\[ \sum M_B = 0 \]

\[ (882 - NB) \]
\[ (400 \times 79°) + (150 \times 4.5 \cos \theta) - (NA \times OA) - (F_A \times DB) = 0 \]
\[ 980 \cos \theta + 675 \times 105.76 - (105.76 \times 9 \sin \theta) - (528.84 \times \cos \theta) = 0 \]
\[ 951.84 \sin \theta + 4759.56 \cos \theta = 3475 \]
\[ \tan \theta = \frac{4759.56}{951.84} \]
\[ \theta = 74° \]
A ladder of weight 200 N is supported on a vertical plane making an angle of 30° with the horizontal. A weight of 1000 N is placed on the top of the ladder as shown in Figure. Determine the minimum value of horizontal force \( P \) to be applied at point B to prevent the ladder from slipping. \((\mu = 0.3)\)

**FBD**

\[
\begin{align*}
\Sigma F_z &= 0, \\
N_A - N_B - P &= 0, \\
N_A - \mu N_B - P &= 0 \\
N_A - 0.3 N_B - P &= 0 \quad (i)
\end{align*}
\]
$$\sum_{ij} = 0.$$ 

$$f_A - M + NB - 1000 = 0.$$ 

$$M 0.3 NA - 200 + NA - 1000 = 0.$$ 

$$\therefore 3NA = 1200.$$ 

$$NA = \frac{1200}{1.8}.$$ 

$$0.3NA + NB = 1200 \quad (2)$$

$$\sum_{MB} = 0.$$ 

$$\left\{ \begin{align*} & (H \times BC) + (1000 \times OB) - (NA \times OA) - (f_A \times OB) = 0. \\
& (400 \times 4.5 \times \frac{13}{2}) + (1000 \times 90 \times \frac{11}{2}) - \left( \frac{1200 - NB \times 9 \times \frac{1}{2}}{0.3} \right) - \\
& \left( 0.3 NA \times 9 \times \frac{13}{2} \right) \\
\end{align*} \right.$$ 

$$6.836 NA = 9339.074$$

$$NA = 1253 \text{ N.}$$

$$NB = 883.18 \text{ N.}$$

$$P = 1006.86 \text{ N.}$$
To predict whether a body subjected to a force is moving or not

1) Assume the body is at rest. Apply eqm conditions
2) Find required ie., frictional force required to maintain the body at rest.
3) If $f_{req} < \mu N$, it is possible case, ie, body is at rest.
   If $f_{req} = \mu N$, the body is in eqm.
   If $f_{req} > \mu N$, it is an impossible case, means body is moving.

A body of weight $100\,\text{N}$ is subjected to a force of $70\,\text{N}$ as shown. If coefficient of friction is 0.4. Predict whether the body is moving or not.

### 1. Body is at rest (assuming)

$\Sigma F_x = 0$

$70 - f = 0$

$f = 70\,\text{N}$

$\mu N$
To find $N$

$\Sigma F_y = 0$

$N - W = 0$

$N = 100 \cdot N$

$\mu N = 0.4 \times 100 = 400 N$

As $F_{eq} > \mu N$, the body is moving.

1. A uniform ladder of weight 800N and of length 7m rest on a horizontal ground and lean against a rough vertical wall. The coefficient of friction btw ladder and floor is 0.3 and btw ladder & vertical wall is 0.2. When a weight of 900N is placed on the ladder at a distance of 2m from the top of the ladder. It is at the point of sliding. Determine

i) The angle made by the ladder with horizontal.

ii) Reaction at the foot of the ladder.

iii) Reaction at the top of the ladder.

2. A uniform ladder of weight 250N and length 5m rest on a horizontal ground and lean against a smooth vertical wall. The angle made by the ladder with the horizontal is 60°. When a man of weight 600N stands on the ladder at a distance of 4m from the top of the ladder. The ladder is at the point of sliding. Determine the coefficient of friction btw the ladder and the floor.
\[ \sum F_y = 0. \]

\[ NA - f_B = 0. \]

\[ NA - 0.3 NB = 0 \quad (1) \]

\[ \sum F_y = 0 \]

\[ f_A + NB - 900 - 800 = 0. \]

\[ 0.2 NA + NB = 1700. \]

\[ 0.2 (0.3 NB) + NB = 1700 \]

\[ 1.06 \ NB = 1700 \]

\[ NB = 1603.77 \text{ N} \]

\[ \overline{FA} = 96.226 \text{ N} \]

\[ \overline{NA} = 481.131 \text{ N} \]

\[ \overline{FB} = 481.131 \text{ N} \]

\[ \sum M_B = 0. \]

\[ (800 \times BD) + (900 \times BC) - (f_A \times OB) - (NA \times DA) = 0. \]

\[ BD = 35° \text{ in } \theta, \quad BC = 50° \text{ in } \theta, \quad OB = 700 \text{ in } \theta, \quad OA = 93 \text{ sin } \theta. \]

\[ (800 \times 3.5 \text{ sin } \theta) + (900 \times 5 \text{ sin } \theta) - (0.2 \times 481.13 \times 700 \text{ sin } \theta) - \]

\[ (481.13 \times 7 \text{ sin } \theta) = 0. \]
\[ 2800 \cos \theta + 4500 \cos \theta = 673.582 \cos \theta = 3367.91 \sin \theta \]

\[ 6626.418 \cos \theta = 3367.91 \sin \theta \]

\[ \tan \theta = \frac{6626.418}{3367.91} = 1.97 \]

\[ \theta = 63.06^\circ \]

\[ \Sigma F_x = 0 \]

\[ NA - FB = 0 \]

\[ NA - MNB = 0 \]

\[ \Sigma F_y = 0 \]

\[ FA + NB - 600 - 250 = 0 \]

\[ NB = 850N \]

\[ NA = 1850. \]

\[ BC = 1 \text{ (0.6)} \]

\[ BD = 2.5 \text{ (0.6)} \]

\[ DA = 5.8 \sin 60^\circ \]

\[ DA = 4.33 \]
\[ \sum M_B = 0. \]
\[ (600 \times BC) + (250 \times BD) - (NA \times DA) = 0 \]
\[ (600 \times 0.5) + (250 \times 1.25) - (\text{some value}) = 0 \]
\[ 3680.5 \mu = 612.5 \]
\[ \mu = 0.17 \]

A body of weight 100N is moving on an inclined plane. A force of 100N is applied on the body in the direction as shown. Determine whether this force is enough to prevent the motion of the body. Given coefficient of friction between body and plane is 0.2.

Assume the body is at rest when it is subjected to 100N force.

\[ \sum F_x = 0. \]
\[ 100 - 300 \sin 30^\circ + f = 0. \]
\[ f = 26 \text{ N} \]
To find $N$

\[ \sum F_y = 0 \]
\[ N - 300 \cos 30^\circ = 0 \]
\[ N = 251.9 \text{ N} \]

\[ \mu N = 0.2 \times 251.9 \]
\[ \mu N = 50.38 \]

Since $F_y < \mu N$, the applied force prevents the motion of block in the downward direction.

<table>
<thead>
<tr>
<th>WEDGE</th>
<th>FRICTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>A wedge is used for raising a heavy object. Angle of wedge is denoted as $\alpha$. And the force applied on the wedge to raise the weight is denoted as $P$. In the figure $P$ is the force applied to lift the load of weight $W$.</td>
<td></td>
</tr>
</tbody>
</table>

FBD of wedge
R₁ is the resultant of F₁, and N₁ acting on surface. 
φ₁ is the angle between R₁ and N₂. R₂ is the resultant of F₂ and N₂. R₁ makes an angle of φ₂ with N₂.

At equilibrium applying equilibrium conditions:

\[ \sum F_x = 0 \]

\[ R_1 \sin \phi_1 + R_3 (\sin \phi_3 + \alpha) - F_2 = 0 \]

\[ \sum F_y = 0 \]

\[ -R_2 \cos (\phi + \alpha) + R_1 \cos \phi_1 = 0 \]

**FBD of weight**

\[ \sum F_x = 0 \]

\[ R_3 \cos \phi_3 - R_2 \sin \alpha + F_2 = 0 \]

\[ \sum F_y = 0 \]

\[ -R_3 \sin \phi_3 + R_2 (\cos \alpha + \phi_2) - W = 0 \]

A load of weight \( W \) is to be lifted using a wedge of angle 10°. Assuming the angle of friction at all surfaces is 15°. Determine the minimum force required to be applied on the wedge.
\[ \Sigma F_x = 0 \]

\[ R_3 \cos \phi_2 - R_2 \sin (\alpha + \phi_2) = 0 \]

\[ R_3 \cos 15^\circ = R_2 \sin 25^\circ \]

0.96 \( R_3 = 0.42 \times R_2 \)

\[ R_3 = 0.44 \times R_2 \quad (1) \]

\[ \Sigma F_y = 0 \]

\[ R_2 \cos (\alpha + \phi_2) - R_3 \sin \phi_3 - \omega = 0 \]

\[ R_2 \cos 25^\circ = R_3 \sin 15^\circ = 10000 \]

0.91 \( R_2 = 0.26 \times R_3 = 10000 \)

0.91 \( R_2 = 0.26 \times 0.44 \times R_2 = 10000 \)

0.79 \( R_2 = 10000 \)

\[ R_2 = 12658.2 \text{ N} \]

\[ R_2 = 0.44 \times 12658.2 \]

\[ = 5569.6 \text{ N} \]
FBD of wedge

\[ \Sigma F_x = 0. \]

\[ R_1 \sin \phi_1 + R_2 \sin (\phi_2 + \alpha) - P = 0. \]

\[ R_1 \sin 15 + R_2 \sin 25 - P = 0. \]

\[ 0.26R_1 + 0.42R_2 = P \quad (1) \]

\[ \Sigma F_y = 0. \]

\[ -R_2 \cos (\phi_2 + \alpha) + R_1 \cos \phi_1 = 0 \]

\[ -R_2 \cos 25 + R_1 \cos 15 = 0 \]

\[ 0.96R_1 = 0.91R_2 \]

\[ R_1 = 0.95R_2 \]

\[ R_1 = 12005.3 \text{ N} \]

\[ P = 8443.02 \text{ N} \]

**Question:** A wedge of 12° is used to lift a weight of 5 ton. If coefficient of friction between all surfaces is 0.2, determine the minimum force required to be applied on the wedge.
FBD of weight

\[ \Sigma F_y = 0 \]
\[ R_3 \cos (\alpha + \phi_2) - R_3 \sin \phi_2 - W = 0 \]
\[ R_3 \cos 23.3 - R_3 \sin 11.3 = 49050 \]
\[ 0.92 R_2 - 0.19 R_3 = 49050 \]
\[ 0.92 R_2 - 0.076 R_2 = 49050 \]
\[ 0.84R_2 = 49050 \]
\[ R_2 = 58392.85 \text{N} \]
\[ R_3 = 20357.14 \text{N} \]

**FBD of wedge**

\[ \Sigma F_x = 0 \]
\[ R_1 \sin \phi_1 + R_2 \sin (\phi_2 + \alpha) - P = 0 \]
\[ R_1 \sin 11.8 + R_2 \sin 22.3 = P \]
\[ 0.19R_1 + 0.39R_2 = P \quad (1) \]

\[ \Sigma F_y = 0 \]
\[ -R_2 \cos (\phi_2 + \alpha) + R_1 \cos \phi_1 = 0 \]
\[ R_1 \cos 11.8 = R_2 \cos 22.3 \]
\[ 0.98R_1 = 0.92R_2 \]
\[ R_1 = 0.94R_2 \]
\[ R_1 = 54889.279 \text{N} \]

\[ P = 33202.17 \text{N} \]
1) A block of weight 1290 N rests on a horizontal surface and suppose another block of weight 570 N as shown in figure. Find the force F applied to the lower block that will be necessary to cause the motion. Take μ as 0.25 between blocks and 0.4 between floor and block.

2) A ladder is 8m long and weighs 200 N. The centre of gravity of the ladder is 3m along the length of ladder from the bottom end. The ladder rests against a vertical wall at B and on the horizontal floor at A. Determine the safe height up to which a man weighing 900 N can climb without making the ladder slip. μ between ladder and floor is 0.4 and μ between ladder and wall is 0.3. The base of the ladder is a distance 5m from the vertical wall.

Determine the angle that ladder makes with horizontal if a painter of 70 kg can climb 4.2m along the ladder without causing the ladder to slip. Length of the ladder is 10m, coefficient of friction is 0.25.
1) A) FBD of (A)

\[ \Sigma F_x = 0 \]
\[ P - f_2 - f_1 = 0 \]
\[ P = 0.25 N_2 - 0.4 N_1 = 0 \] \( \cdots (1) \)

\[ \Sigma F_y = 0 \]
\[ N_1 - H - N_2 = 0 \]
\[ N_1 - N_2 = 1270 \] \( \cdots (2) \)

\[ N_1 = 1770 N \]
\[ P = 588 \, 928 N \]

\[ f_2 = T \cos(36.9) = 0 \]
\[ 0.25 N_2 - 0.4 B = 0 \] \( \cdots (3) \)

\[ \Sigma F_y = 0 \]
\[ N_2 + T \sin(36.9) = 0 \]
\[ N_2 + 0.69 = 570 \] \( \cdots (4) \)

\[ N_2 = 480N \]
\[ T = 150 N \]

(continued on next page...)

2) A

\[ \cos^{-1} \left( \frac{5}{8} \right) = 51.3^\circ \]
\[ \sum F_x = 0. \]
\[ N_B - F_A = 0. \]
\[ N_B = 0.4 \times N_A \]
\[ \sum F_y = 0 \]
\[ F_B - 300 - 900 + N_A = 0. \]
\[ 0.3N_B + N_A = 1200. \]
\[ 0.3(0.4N_A) + N_A = 1200 \]
\[ 1.12N_A = 1200 \]
\[ N_A = 1071.43 \text{ N} \]
\[ N_B = 428.57 \text{ N} \]
\[ F_A = 428.572 \text{ N} \]
\[ F_B = 188.571 \text{ N} \]

\[ \sum M_A = 0. \]
\[ (300 \times AC) + (900 \times AD) - (F_B \times OA) - (N_B \times OB) = 0. \]
\[ AC = 300 \times 51.3 \]
\[ AD = 900 \times 51.3 \]
\[ OA = 5 \text{ cm} \]
\[ OB = 6.2 \text{ cm} \]
\[ AC = 1.9 \text{ cm} \]
\[ AD = 0.62x \]

\[ (300 \times 1.9) + (900 \times 0.62x) - (128.571 \times 5) - (428.572 \times 6.2) = 0. \]
\[ 558x = 9730.0014 \]
\[ x = 4.9 \text{ m} \]
\[ \Sigma F_x = 0 \]
\[ N_A - F_B = 0 \]
\[ N_A = 1.25 N_B \]
\[ N_A = 0.25 N_B \quad \text{(1)} \]

\[ \Sigma F_y = 0 \]
\[ F_A - 686.7 - 0.25 N_B = 0 \]
\[ 0.25 (0.25 N_B) - 686.7 + N_B = 0 \]
\[ N_B = \frac{686.7}{1.06} = 647.83 \text{ N} \]
\[ N_A = 161.95 \text{ N} \]
\[ \Sigma M_B = 0 \]
\[ 24 (686.7 \times BC) - (F_A \times OB) - (N_A \times OA) = 0 \]
\[ (686.7 \times 4.2 \cos \theta) - (40.49 \times 10 \cos \theta) - (161.95 \times 10 \sin \theta) = 0 \]
\[ 2479.24 \cos \theta = 1619.5 \sin \theta \]
\[ \tan \theta = 1.53 \]
\[ \theta = 56.85^\circ \]
PRINCIPLE OF VIRTUAL WORK

Definition of work is given by \( W = F \cdot s \), where 'F' is the force applied and 's' is the displacement of the body. \( W = F_s \cos \theta \) when 'θ' is the angle between force and displacement. Work can be positive, zero or negative. Work is said to be zero, when displacement is perpendicular to the force applied. Work is negative when displacement is opposite to the applied force.

VIRTUAL DISPLACEMENT

It is a small imaginary displacement assumed in a body or system of bodies which is consistent with the geometrical conditions.

VIRTUAL WORK

The work done by a force due to virtual displacement is called virtual work.

PRINCIPLE OF VIRTUAL WORK

It states that when a force or a system of forces acting on a body or system of bodies is in eqm the algebraic sum of virtual work by the system of forces, acting on the body is equal to zero.

A beam of length 5 m is subjected to a force of 10 N as shown in figure. Determine the reaction at A and B.
Assuming a virtual displacement of $\Delta y_B$ at point B. Position of B is shown in figure.

Virtual work due to $R_A = R_A \times 0 = 0$

Virtual work due to $10N = (10 \Delta y_C) \text{Nm}$

Virtual work due to $R_B = -(R_B \Delta y_B)$ (Note indicates force-f displacement in opposite directions).

By principle of virtual work, algebraic sum of virtual work:

$$0 + 10 \Delta y_C - R_B \Delta y_B = 0.$$

From figure we have $\frac{\Delta y_C}{\Delta y_B} = \frac{AC}{AB} = \frac{2}{5}$

$\Delta y_C = \frac{2}{5} \Delta y_B = 0.4 \Delta y_B.$

$$10(0.4 \Delta y_B) - R_B \Delta y_B = 0.$$

$$R_B = 4$$

we have $R_A + R_B = 10$

$\therefore R_A = 6$
A beam of span 12 m is shown in figure. Determine the reaction on its ends using principle of virtual work.

A. Imagine a virtual displacement of \( \Delta y_B \) at point B.

Virtual work due to \( R_A = R_A \times 0 = 0 \).

\[ 20kN = 20 \times \Delta y_C \text{ kNm} \]

\[ 10kN = (10 \Delta y_D) \text{ kNm} \]

\[ R_B = (R_B \Delta y_B) \]

\[ \frac{\Delta y_C}{\Delta y_D} = \frac{2}{5}, \quad \frac{\Delta y_C}{\Delta y_B} = \frac{2}{12}. \]

\[ \Delta y_D = 0.4 \Delta y_C, \quad \Delta y_B = 0.17 \Delta y_C. \]

\[ 20 \Delta y_C + 10 \times 0.4 \Delta y_C - \frac{R_B \times 0.17 \Delta y_C}{0.4} = 0 \]

\[ R_B = 144,138 \text{ N} = 7,851 \text{ N}. \]

\[ R_A = 30 - 7.65 \]

\[ = 22.35 \text{ N} \]
Determine the horizontal force \( P \) necessary to push a 1000 N roller over the 15 cm obstruction as shown in figure.

2) Two spheres each of weight 500 N and of radius 100 mm rest in a horizontal channel of width 360 mm as shown in figure. Find the reactions at all points. Assume all the surfaces of contact are smooth.

3) Two cylinders of diameter 50 mm and 85 mm weighing 150 N and 50 N respectively are placed as shown in figure. Assuming all contact surfaces are smooth, find the reactions at A, B, C, and D.
FBD of ball

Since there are three forces acting on the ball and it is in eqn. All three forces must pass through a single point, i.e., centre point.

\[ \theta = \cos^{-1} \left( \frac{15}{30} \right) = 60^\circ \]

\[ \Sigma F_x = 0. \\
\]

\[ P - R \sin 60^\circ = 0. \\
P - R \frac{18}{2} = 0 \\
\Sigma F_y = 0 \\
R \cos 60^\circ - \mu = 0 \\
R 0.5 = 1000 \\
R = 2000 \text{ N} \\
P = 1732.05 \text{ N} \]

\[ 2) \ A \]

\[ \Sigma F_x = 0 \\
R_A - R_D \cos 36.8^\circ = 0. \\
R_A = 0.8 R_D \quad (1) \]

\[ P_R = 360 - 60 - 0.0 \\
P_R = 160 \text{ N} \\
P_Q = 200 \text{ N} \]

\[ \theta = \cos^{-1} \left( \frac{180}{200} \right) \\
= 68.4^\circ \text{ or } 36.8^\circ \]
\[ \Sigma F_y = 0 \]
\[ R_B - W - R_D \sin 36.8^\circ = 0 \]
\[ R_B = 0.6 R_D = 500 \] \( \text{--- (2)} \)

\[ \Sigma F_x = 0 \]
\[ -R_C + R_D \cos 36.8^\circ = 0 \]
\[ 0.8 R_D = R_C \]
\[ \Sigma F_y = 0 \]
\[ R_D \sin 36.8^\circ - W = 0 \]
\[ 0.6 R_D = 500 \]
\[ R_D = 833.33 \text{ N} \]
\[ R_C = 666.664 \text{ N} \]
\[ R_B = 999.998 \text{ N} \]
\[ R_A = 666.664 \text{ N} \]

3) A \ FBD of (2) 

\[ \theta = \cos^{-1} \left( \frac{500}{833.33} \right) \]
\[ = 23.07^\circ \]
\[ \sum F_x = 0 \]
\[ R_A - R_D \cos 23.07 - R_B \cos 45^\circ = 0 \]
\[ R_A = 0.92 R_D - 0.707 R_B = 0 \quad (1) \]

\[ \sum F_y = 0 \]
\[ -R_D \sin 23.07 + R_B \sin 45^\circ = 150 \]
\[ 0.707 R_B - 0.39 R_D = 150 \quad (2) \]

\[ \overbrace{FBD \ of \ \text{A}} \]

\[ \sum F_x = 0 \]
\[ R_C + R_D \cos 23.07 = 0 \]
\[ R_C = 0.92 R_D \quad (3) \]

\[ \sum F_y = 0 \]
\[ R_D \sin 23.07 = 50 \]
\[ R_D = 128.203 \text{ N} \]
\[ R_C = 117.95 \text{ N} \]
\[ R_B = 286.72 \text{ N} \]
\[ R_A = 3167.947 \text{ N} \]